Self-Assessment for Grade 12 College Technology Math (MCT4C)

Students who are registered for Grade 12 College Technology Math (MCT4C) may benefit from a self evaluation and review of the following sample of expectations from Grade 11 University/College Math (MCF3M).

The questions in this self-assessment reflect some of the key ideas learned in prerequisite courses. They do not represent the problem solving approach or the rich experience that students would be exposed to in a classroom. The intention is for students to revisit some key concepts and, if needed, access review materials in an informal environment at a pace that is comfortable for the student.

Concept	**Sample Question and Answer**	**How comfortable do you feel with this concept?**	**Link for further support**
I can solve quadratic equations	1. Solve by factoring a) $x^2 - x - 2 = 0$ b) $-14x^2 + 7x = 0$ c) $x^2 - 81 = 0$	 ☐ **Very comfortable**  ☐ **Somewhat comfortable**  ☐ **Not at all comfortable**	[Solving Quadratic Equations]

I can determine the real roots of a variety of quadratic equations (i.e., graphing; factoring; using the quadratic formula)	2. Determine the roots of each quadratic. a) $x^2 + 6x + 8 = 0$ b) $3x + 3 = 5x^2$	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	Introduction to the Quadratic Formula
I can sketch graphs of $g(x) = a(x-h)^2 +$ by applying one or more transformations to the graph of $f(x) = x^2$	3. For the quadratic relation $y = -(x+3)^2 + 4$ a) Graph the Relation State the: b) direction of the opening; c) coordinates of the vertex; d) equation of the axis of symmetry; e) y-intercept; f) x-intercepts.	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	Transformations of Parabolas

I can sketch graphs of quadratic functions in the factored form $f(x) = a(x - r)(x -$ by using the x-intercepts to determine the vertex	4. Sketch the function $f(x) = 2(x-1)(x-5)$	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	[Graphing and Equations in Factored Form]
I can solve problems arising from real-world applications, given the algebraic representation of a quadratic function.	5. Given the equation of a quadratic function representing the height of a ball over elapsed time $h(t) = -0.75t^2 + 15t$ a) Determine the maximum height of the ball. b) Determine the length of time needed for the ball to touch the ground. c) Determine the time interval when the ball is higher than 40m.	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	[Quadratic Applications]

I can evaluate, with and without technology, numerical expressions containing integer and rational exponents and rational bases	6. Evaluate a) $\left(2^{-1}\right)^{2}$ b) $(-27)^{\frac{1}{3}}$ c) $\left(\frac{25}{4}\right)^{\frac{3}{2}}$	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	Exponent Laws all Together
I can describe key properties for exponential functions (domain and range, intercepts, increasing/decreasing intervals, and asymptotes)	7. Describe key properties for $f(x) = 200\left(\frac{1}{2}\right)^{x}$	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	Properties of Basic Exponential Functions

I can graph, with and without technology, an exponential relation, given its equation in the form $y = a^x, \{a > 0, a \neq 1\}$	8. Graph the function $$f(x) = 200\left(\frac{1}{2}\right)^x$$	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	[Transformations of Exponential Functions]
I can solve problems using given graphs or equations of exponential functions arising from a variety of real-world applications by interpreting the graphs	9. New Zealand has a population of about 6 million and it is estimated that the population will double in 36 years, modelled by $P(t) = P_0(2)^{\frac{t}{36}}$. If population growth remains the same, what will the population be in 15 years? 40 years?	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	[Modelling with Exponential Functions]

I can solve real-world application problems by using the primary trigonometric ratios, by determining the measures of the sides and angles of right triangles	10. To measure the height of a tall tree, a forester paces 40 m from the base of the tree and measures an angle of elevation to the top of the tree. If the angle is $65°$, determine the height of the cedar tree.	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	Sine and Cosine Ratios
I can solve problems that require the use of the sine law or the cosine law in acute triangles.	11. Two ships left Port Huron on Lake Ontario at the same time. One travelled at 12 km/h on a course of $235°$. The other travelled at 15km/h on a course of $105°$. How far apart are they four hours later?	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	The Sine Law

I can sketch graphs of $f(x) = a\sin(x - d) +$ by applying the transformation to the graph of $f(x) = \sin x$ and determine and describe its key properties (i.e., cycle, domain, range, intercepts, amplitude, period, maximum and minimum values, increasing/ decreasing intervals)	12. Consider $f(x) = 0.5\sin(x - 30) + 1$ a) Sketch the graph over the interval $0^o \leq x \leq 360^o$. b) Describe its key properties.	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	[Investigate Transformations of Sinusoidal Functions]
I can identify periodic and sinusoidal functions	13. Which of the following scenarios could be modelled by periodic functions? a) Sunset times in Aurora, Ontario. b) The motion of a planet around the sun. c) A child's movement on a swing. d) The growth of a bacteria.	☐ **Very comfortable** ☐ **Somewhat comfortable** ☐ **Not at all comfortable**	[Modelling Periodic Behaviour]

Solutions to Sample Questions

1. Solve by factoring

a) $x^2 - x - 2 = 0$
b) $-14x^2 + 7x = 0$
c) $x^2 - 81 = 0$

Solutions:

a) $(x-2)(x+1) = 0 \rightarrow x - 2 = 0 \; or \; x + 1 = 0 \rightarrow x = 2, -1$

b) $7x(-2x+1) = 0 \rightarrow x = 0 \; or \; -2x + 1 = 0 \rightarrow x = 0, \frac{1}{2}$

c) $(x-9)(x+9) = 0 \rightarrow x = 9, -9$

2. Determine the roots of each quadratic.
a) $x^2 + 6x + 8 = 0$
b) $3x + 3 = 5x^2$

Solutions:
a) $(x+4)(x+2) = 0 \rightarrow x = -4, -2$
b)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(-3)}}{2(5)}$$

$$x = \frac{3 \pm \sqrt{9 + 60}}{10}$$

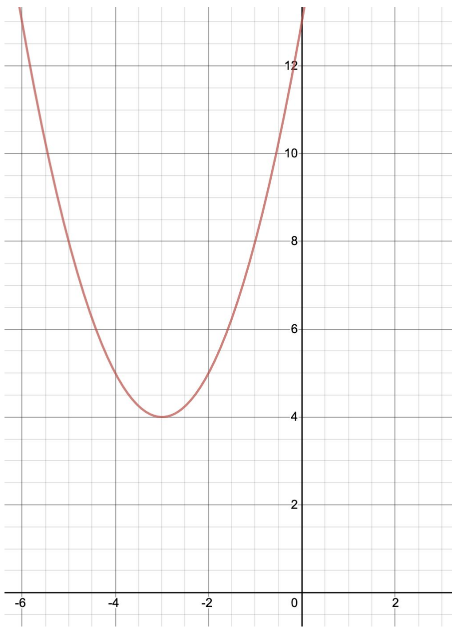
$$x = -0.53, \; 1.13$$

3. For the quadratic relation $y = -(x+3)^2 + 4$
a) Graph the Relation

State the:
b) direction of the opening;
c) coordinates of the vertex;
d) equation of the axis of symmetry;
e) y-intercept;
f) x-intercepts.

Solutions:

a)



b) up

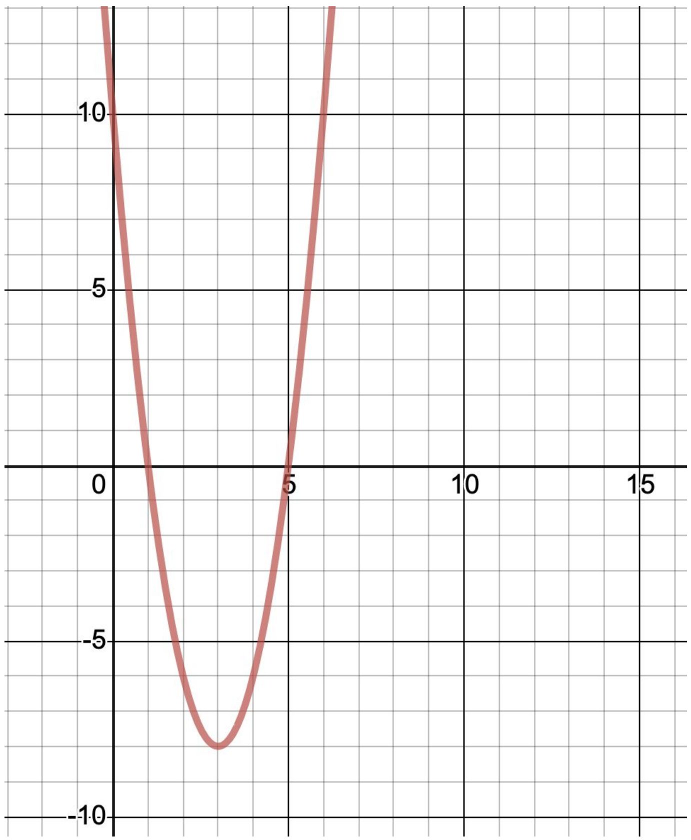
c) $(-3, 4)$

d) $x = -3$
e) $(0,\ 13)$
f) there are no x-intercepts for this function.

4. Sketch the function
$f(x) = 2(x-1)(x-5)$

Solution:

Use the x-intercepts to plot. Sketch the axis of symmetry as $x = 3$**. Locate the vertex using substitution of** $x = 3 \rightarrow f(3) = -8$**. Connect intercepts and vertex to sketch.**



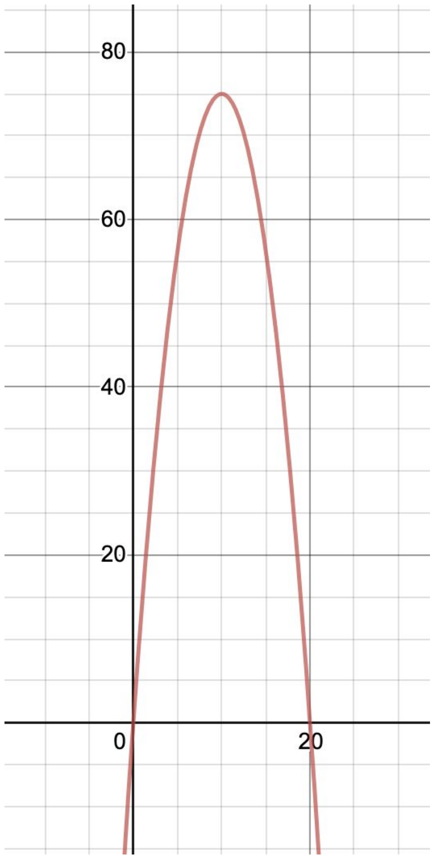
5. Given the equation of a quadratic function representing the height of a ball over elapsed time $h(t) = -0.75t^2 + 15t$

a) Determine the maximum height of the ball.

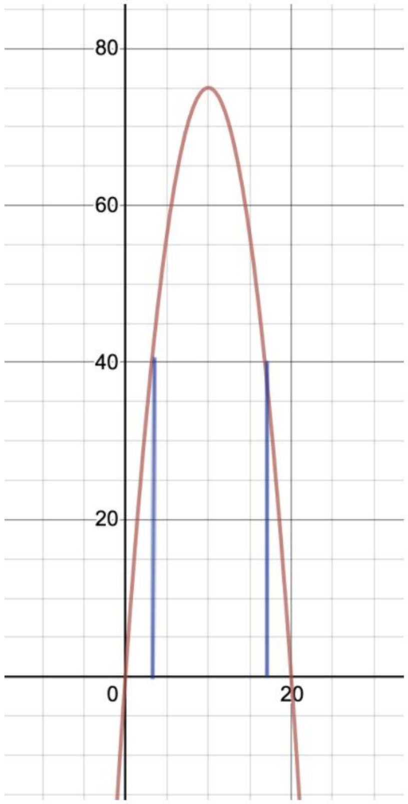
b) Determine the length of time needed for the ball to touch the ground.

c) Determine the time interval when the ball is higher than 40m.

Solutions:



a) Max height 75m
b) 20 seconds
c)



Using interpolation - it appears that the height is greater than 40m from approximately 3 seconds to approximately 17 seconds, for a total of approximately 14 seconds.

Alternatively, one could also solve the equation $-0.75t2+15t=40$ **to get 13.7 seconds.**

6. Evaluate

a) $\left(2^{-1}\right)^{2}$

b) $(-27)^{\frac{1}{3}}$

c) $\left(\frac{25}{4}\right)^{\frac{3}{2}}$

Solutions:

a) $2^{-2} = \left(\frac{2}{1}\right)^{-2} = \left(\frac{1}{2}\right)^{2} = \frac{1}{4}$

b) $\sqrt[3]{-27} = -3$

c) $\left(\frac{25}{4}\right)^{\frac{3}{2}} = \sqrt[2]{\left(\frac{25}{4}\right)^{3}} = \left(\frac{5}{2}\right)^{3} = \frac{125}{8}$ **or** $\left(\frac{25}{4}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{25}{4}}\right)^{3} = \left(\frac{5}{2}\right)^{3} = \frac{125}{8}$

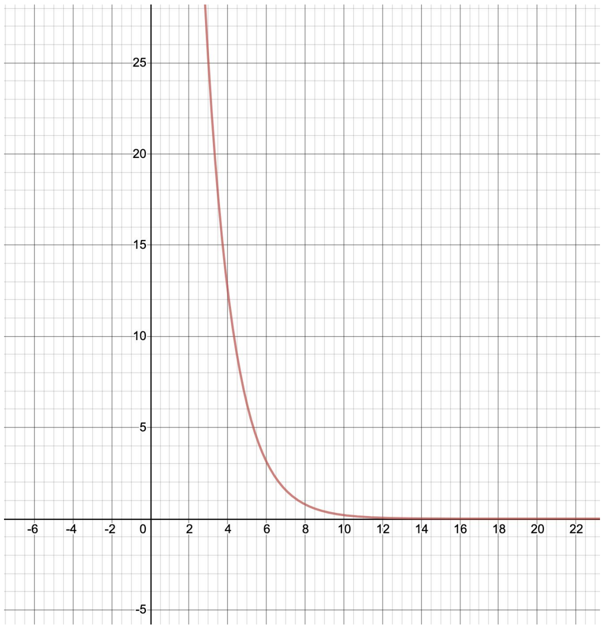
7. Describe key properties for $f(x) = 200\left(\frac{1}{2}\right)^{x}$

Solution:

decay curve.
decreasing function .
Rate of decay = $\frac{1}{2}$ **.**
y-intercept at 200.

8. Graph the function $f(x) = 200\left(\frac{1}{2}\right)^x$

Solution:



9. New Zealand has a population of about 6 million and it is estimated that the population will double in 36 years, modelled by $P(t) = P_0(2)^{\frac{t}{36}}$. If population growth remains the same, what will the population be in 15 years? 40 years?

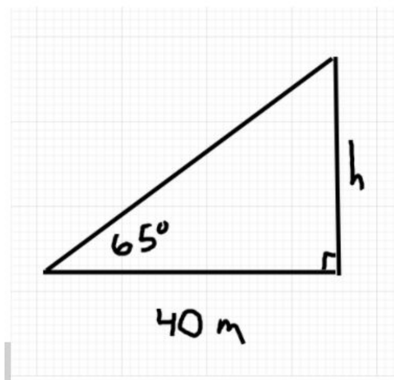
Solutions:

a) $P(t) = 6(2)^{\frac{15}{36}} = 8.009$ **million**

b) $P(t) = 6(2)^{\frac{40}{36}} = 12.9607$ **million**

10. To measure the height of a tall tree, a forester paces 40 m from the base of the tree and measures an angle of elevation to the top of the tree. If the angle is $65°$, determine the height of the cedar tree.

Solution:



$$\tan 65° = \frac{h}{40}$$

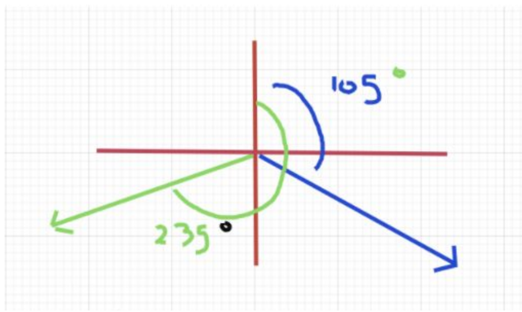
$$h = 40\tan 65°$$

$$h = 86$$

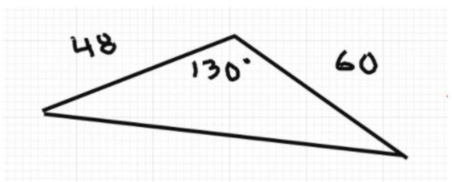
Therefore the height of the tree is 86m.

11. Two ships left Port Huron on Lake Ontario at the same time. One travelled at 12 km/h on a course of $235°$. The other travelled at 15km/h on a course of $105°$. How far apart are they four hours later?

Solution:



From the diagram, the obtuse angle formed between the directions of the two ships is $130°$.
After four hours of travel, the ships have travelled respectively 48 km and 60km.



$$a^2 = 48^2 + 60^2 - 2(48)(60)\cos 130^o$$
$$= 9606.45$$
$$a = 98$$

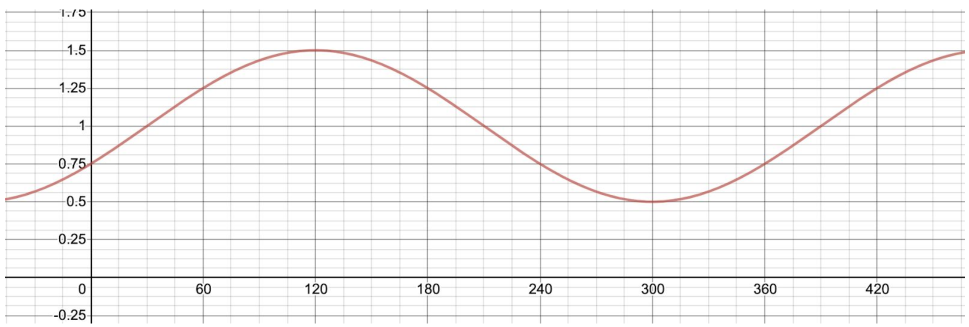
Therefore the two ships are 98km apart after 4 hours.

12. Consider
$f(x) = 0.5\sin(x - 30) + 1$

a) Sketch the graph over the interval $0° \leq x \leq 360°$.

b) Describe its key properties.

Solutions:

a) 

b) Amplitude = 0.5. Period length = 360° **. Phase shift of** 30° **right. Vertical translation of 1 unit up. Maximum at 1.5. Minimum at 0.5.**

13. Which of the following scenarios could be modelled by periodic functions?

a) Sunset times in Aurora, Ontario

b) The motion of a planet around the sun

c) A child's movement on a swing

d) The growth of a bacteria

Solutions:

a and b only.