

Self-Assessment for Grade 12 Calculus and Vectors (MCV4U)

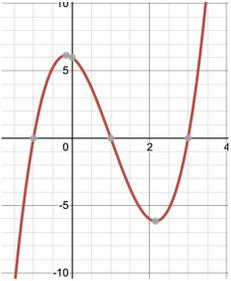
Students who are registered for Grade 12 Calculus and Vectors (MCV4U) may benefit from a self evaluation and review of the following sample of expectations from Grade12 Advanced Functions (MHF4U).

The questions in this self-assessment reflect some of the key ideas learned in prerequisite courses. They do not represent the problem solving approach or the rich experience that students would be exposed to in a classroom. The intention is for students to revisit some key concepts and, if needed, access review materials in an informal environment at a pace that is comfortable for the student.

Concept(s)	Sample Question	How comfortable do you feel with this concept?	Link(s) to explore concept further
I can use the laws of logarithms to simplify and evaluate numerical expressions	1. Evaluate. a) $\log_2 16^3$ b) $\log_2 8^{0.5}$ c) $2\log 5 + \frac{1}{2}\log 16$ d) $\log_3 405 - \log_3 5$	<div style="display: flex; flex-direction: column; align-items: center;"> <div style="display: flex; align-items: center; margin-bottom: 10px;">  <input data-bbox="1503 857 1541 893" style="margin-right: 10px;" type="checkbox"/> Very comfortable </div> <div style="display: flex; align-items: center; margin-bottom: 10px;">  <input data-bbox="1503 971 1541 1006" style="margin-right: 10px;" type="checkbox"/> Somewhat comfortable </div> <div style="display: flex; align-items: center;">  <input data-bbox="1503 1091 1541 1127" style="margin-right: 10px;" type="checkbox"/> Not at all comfortable </div> </div>	Properties of Logs

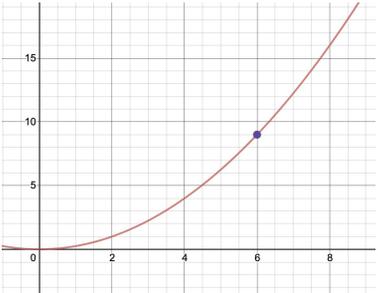
<p>I can recognize the relationship between an exponential function and the corresponding logarithmic function to be that of a function and its inverse,</p>	<p>2. Is the inverse of $y=2^x$ a function? Explain your answer using algebraic and graphical reasoning.</p>	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Introduction to Logarithms Video</p>
<p>I can describe the roles of the parameters a, k, d and c in terms of transformations on the graph of $f(x) = \log_{10}x$</p>	<p>3. If $f(x) = \log_3x$ sketch the graph of $g(x) = 2f(2(x+1)) - 3$</p>	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Logarithmic Functions - Part 5 Video</p>

<p>I can solve exponential and logarithmic equations</p>	<p>4. Solve.</p> <p>a) $5^t = 250$</p> <p>b) $4^{x+5} = 64^x$</p> <p>c) $\log_4(x+2) = 3$</p>	<p> <input type="checkbox"/> Very comfortable</p> <p> <input type="checkbox"/> Somewhat comfortable</p> <p> <input type="checkbox"/> Not at all comfortable</p>	<p>Solving Exponential Equations Video</p>
<p>I can solve linear and quadratic trigonometric equations, with and without graphing technology, for the domain of real values from 0 to 2π</p>	<p>5. Solve the equation $2\cos^2x = \sin x + 1$ over the interval $0 \leq x \leq 2\pi$. Leave your answers as exact values.</p>	<p> <input type="checkbox"/> Very comfortable</p> <p> <input type="checkbox"/> Somewhat comfortable</p> <p> <input type="checkbox"/> Not at all comfortable</p>	<p>Introduction to Trigonometric Ratios and Special Triangles Video</p> <p>Solving Trigonometric Equations Video</p>

<p>I can sketch graphs of $y = a \sin(k(x - d)) + c$ and $y = a \cos(k(x - d)) + c$ by applying transformations to the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ with angles expressed in radians</p>	<p>6. If $f(x) = \sin(x)$, sketch the graph of $g(x) = -f\left(2\left(x + \frac{\pi}{6}\right)\right) - 1$</p>	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Graphs of Primary Trigonometric Functions Video</p>
<p>I can describe key features of the graphs of polynomial functions</p>	<p>7. Describe key features of the graph of $y = f(x)$ given below. (End behaviour, intervals of increase and decrease, and where $f(x) > 0$)</p> 	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Characteristics of Polynomial Functions Video</p>

<p>I can sketch the graph of a polynomial function given in factored form using its key features</p>	<p>8. Sketch the graph of $f(x) = -2(x+3)^2(x-1)$</p>	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Graphs of Polynomial Functions in Factored Form Video</p>
<p>I can sketch the graph of a simple rational function using its key features, given the algebraic representation of the function</p>	<p>9. Sketch the graph of $f(x) = \frac{2x+1}{x+2}$</p>	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Rational Functions of the Form $y = \frac{ax+b}{cx+d}$</p>

<p>I can make connections between instantaneous rates of change and average rates of change</p>	<p>10. Describe how the average rate of change can be used to determine the instantaneous rate of change in any function.</p>	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Determining Average and Approximating Instantaneous Rates of Change for Linear, Polynomial and Rational Functions Video</p>
<p>I can make connections, between the slope of a secant on the graph of a function and the average rate of change of the function over an interval</p>	<p>11. Determine the average rate of change graphically and algebraically for the function $f(x) = 2x^2 - 5$ from $x=1$ to $x=2$.</p>	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p>Determining Average and Approximating Instantaneous Rates of Change for Linear, Polynomial and Rational Functions Part A Video</p>

<p>I can determine the approximate slope of the tangent to a given point on the graph of a function and make connections to average and instantaneous rates of change</p>	<p>12. Determine an approximation for the instantaneous rate of change using two different methods for the below function, at the point where $x = 1$.</p> 	<p>  <input type="checkbox"/> Very comfortable  <input type="checkbox"/> Somewhat comfortable  <input type="checkbox"/> Not at all comfortable </p>	<p> Determining Average and Approximating Instantaneous Rates of Change for Linear, Polynomial and Rational Functions More Golf Video Generalizing Our Results: Determining Average and Approximating Instantaneous Rates of Change for Linear Polynomial and Rational Functions Video </p>
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Solutions to Sample Questions

1. Evaluate.

a) $\log_2 16^3 = 12$

b) $\log_2 8^{0.5} = 1.5$

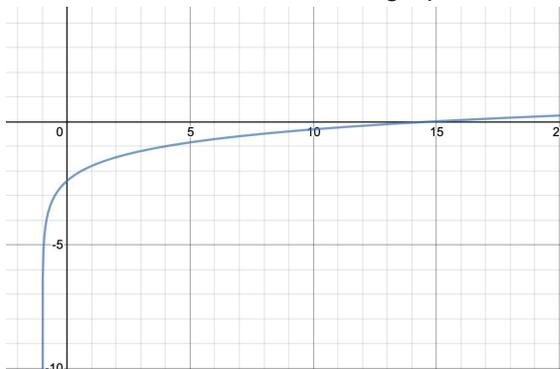
$$\text{c) } 2\log 5 + \frac{1}{2}\log 16 = \log 5^2 + \log 16^{\frac{1}{2}} = \log(25 \times 4) = \log 100 = 2$$

$$\begin{aligned} \text{d) } & \log_3 405 - \log_3 5 \\ & = \log_3 \left(\frac{405}{5} \right) = \log_3(81) = 4 \end{aligned}$$

2. Is the inverse of $y = 2^x$ a function? Explain your answer using algebraic and graphical reasoning.

Yes. For every x there is only one y -value. The function also passes the vertical line test.

3. If $f(x) = \log_3 x$ sketch the graph of $g(x) = 2f(2(x+1)) - 3$



4. Solve.

$$\text{a) } 5^t = 250$$

$$t = \log_5 250 = 3.43$$

b) $4^{x+5} = 64^x$

$$4^{x+5} = 4^{3x}$$

$$x + 5 = 3x$$

$$x = 2.5$$

c) $\log_4(x + 2) = 3$

$$x + 2 = 4^3$$

$$x + 2 = 64$$

$$x = 62$$

5. Solve the equation $2\cos^2x = \sin x + 1$ over the interval $0 \leq x \leq 2\pi$. Leave your answers as exact values.

$$2\cos^2x - \sin x - 1 = 0$$

$$2(1 - \sin^2x) - \sin x - 1 = 0$$

$$2\sin^2x + \sin x - 1 = 0$$

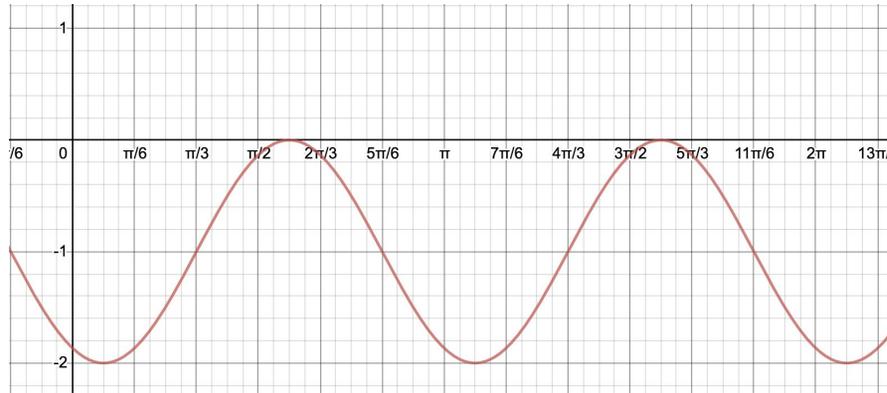
$$(\sin x + 1)(2\sin x - 1) = 0$$

$$\sin x = -1 \mapsto x = \frac{3\pi}{2}$$

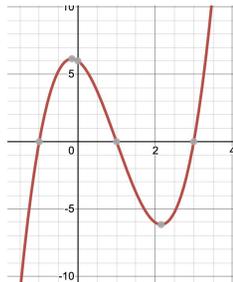
$$\sin x = \frac{1}{2} \mapsto x = \frac{\pi}{6}, \frac{5\pi}{6}$$

6. If $f(x) = \sin(x)$, sketch the graph of

$$g(x) = -f\left(2\left(x + \frac{\pi}{6}\right)\right) - 1$$

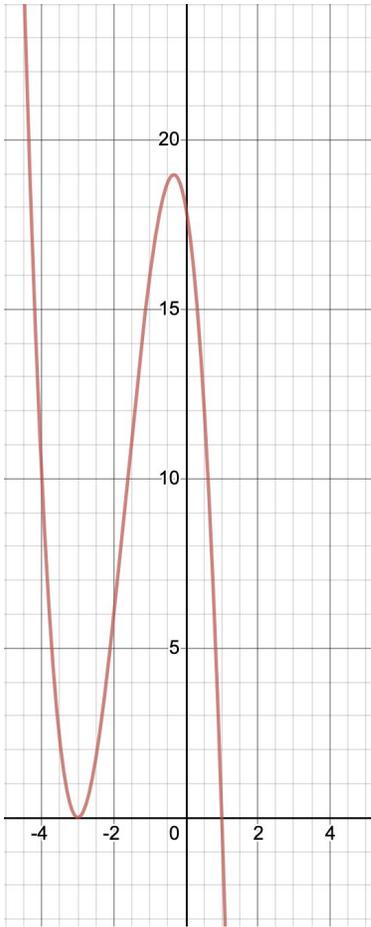


7. Describe key features of the graph of $y = f(x)$ given below. (End behaviour, intervals of increase and decrease, and where $f(x) > 0$)



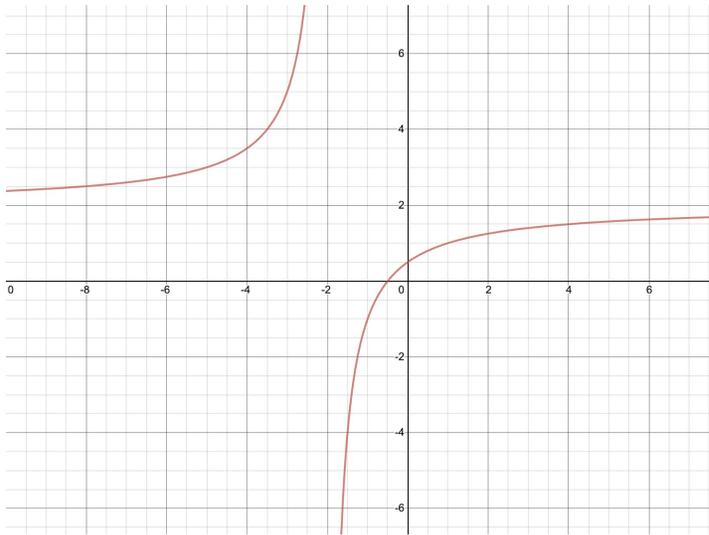
- odd degree
- 3rd degree polynomial
- positive leading coefficient
- x- intercepts at $x = -1, 1, 3$
- y-intercepts at $y = 6$
- Begins in quad III and ends in quad I

8. Sketch the graph of $f(x) = -2(x + 3)^2(x - 1)$



9. Sketch the graph of

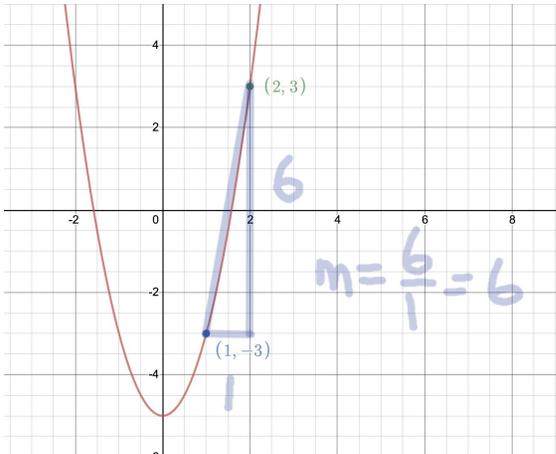
$$f(x) = \frac{2x+1}{x+2}$$



10. Describe how the average rate of change can be used to determine the instantaneous rate of change in any function.

Select two points very close to one another, e.g. a and $a + 0.0001$, to approximate the instantaneous rate of change.

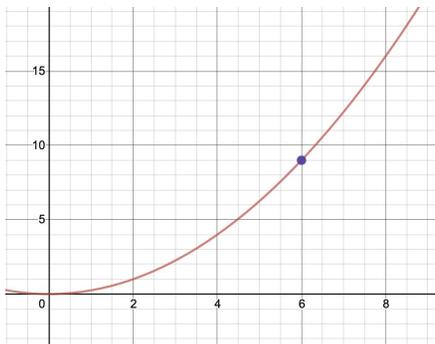
11. Determine the average rate of change graphically and algebraically for the function $f(x) = 2x^2 - 5$ from $x=1$ to $x=2$.



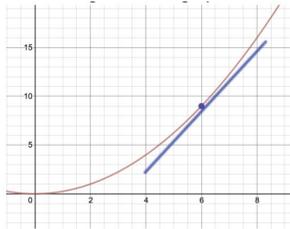
Ave rate of change:

$$\frac{3 - (-3)}{2 - 1} = 6$$

12. Determine an approximation for the instantaneous rate of change using two different methods for the below function, at the point where $x=6$.



Draw a tangent on the graph and determine the slope of the line which is the instantaneous rate of change
 $m =$



$$m = \frac{8}{3}$$

Determine the equation of the function $\rightarrow y = 0.25x^2$

$$\text{iroc} = \frac{0.25(6)^2 - 0.25(6.001)^2}{6 - 6.001} = 3$$