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USING TECHNOLOGICAL COMMUNICATION TOOLS FOR TEACHING MATHEMATICS TO ELEMENTARY STUDENTS TO ENHANCE THEIR LEARNING EXPERIENCE

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This article reports the findings of a study investigating the merits or otherwise of using common technological tools to teach Mathematics concepts to elementary students, particularly grade 7 and 8 students, in a North American school context. It is widely believed that Mathematics is a traditional subject that is best taught and learned in a brick and mortar classroom and that the teacher's role is essentially to impart knowledge to students. However, in this 21st century, school-aged students are said to have been born in the technology era and are more than ever connected. Whilst many students still prefer to be in a physical classroom to learn, as evidenced by the number of students taking online courses compared to those selecting in-class courses in most educational settings, extant research shows that the advent of technology and the internet have undeniably revolutionized the way students, particularly teenagers with their ease to use internet accessible devices, learn nowadays. High school and college students are increasingly opting to study online due to various reasons and consequently, leading to a corresponding increase in the number of school boards and universities in North America offering online courses. Hence, our article focuses on the usefulness of technological tools and associated factors that keep students engaged for learning purposes, with particular reference to the enhancement of their mathematical knowledge and skills via online learning. Whist confirming the findings of other researchers in the field, various other aspects pertaining to how students better learn through online interactions, depicted in our study, are reported and discussed at length. Significantly the paper identifies new perspectives worthy of consideration when designing a fully online teaching and learning quality environment with teenagers as the targeted audience.

Keywords: Online learning, Mathematics education, Web 2.0, 21st century learning, Educational technology, Technological communication tools.

Introduction

Technology is omnipresent and is permeating all spheres of our society. Looking back some decades ago, technological devices were used only by those who could afford the luxury of having a mobile phone that was too big to fit into one's pocket. Nowadays, the availability and ease to acquire such smart device that allows instant communication via SMS (Short Messaging Service), MMS (Multimedia Messaging

Service), or access to a network at the tap of a button is no longer a myth but a reality. More and more powerful technological devices are being manufactured and thus, bringing in new ways of teaching and learning, particularly in schools. Teenagers and school-aged students of this 21st century have been born in the technological era and many own at least one internet capable gadget. Consequently, this technological influx has led to an increase in blended and fully online learning environments as evidenced by the number of students taking online courses. "Over 1 million students took online courses in the 2007-08 school year, and it is estimated that five million students (i.e. 10% of K-12 students) will take online courses in the next five years in the USA" (Kim, Park & Cozart, 2014, p. 172). School boards throughout North America, mostly in high schools, are offering many courses through online learning. They also offer professional development courses to their teaching and non-teaching staff which can be a very practical and cost-effective initiative for busy adults. It is not a coincidence that the National Council of Teachers of Mathematics (NCTM) (2008) has research questions related to technology. Two out of the ten research-guiding questions that originated from researchers and practitioners of mathematics focus on the integration of technology in the school classroom.

More and more courses are being offered through online learning nowadays. This gradual shift from the traditional brick and mortar classroom to a more collaborative, constructivist and connected online learning approach has given rise to the prevalence of computer-based instruction as a means of meeting student needs and differentiating instruction through activities that include tutorials, drill-and-practice, and assessment of student knowledge (Martindale *et al*., 2005). Some people prefer this mode of learning because of the flexibility that it offers in terms of delivery and scheduling as one does not necessarily need to be at a specific place at a specific time. The learner may have a full time occupation during the day and still study in the evening and/or weekend. Moreover, online learning may sometimes be more effective for some type of learners; for example, those who do not like to share openly in front of everyone, or those who need time to think before sharing their answer. Do online courses work with students in the intermediate grades (grades 7 and 8)? Is online learning comparable to or better than in-class learning for elementary students learning Mathematics? This article aims to shed some light on the various elements pertaining to learning Mathematics through a fully online learning environment for elementary students. It approaches the question through both perspectives of a grade 7/8 student and of a teacher/facilitator.

Literature Review: A General Overview

There are various factors that are generally taken into consideration when planning and designing a fully online technology-enabled math learning environment. The NCTM (2008, p. 23) defines a 'technology-enabled learning setting' as "an environment in which teaching and learning occur through and with the aid of electronic forms of technology, with full recognition of the presence and impact of manipulative and other nonelectronic technologies on student learning." It may have mathematical technologies and/or communicative and collaborative technologies where the latter allow users to "create, manipulate, edit, communicate, and share experiences, ideas, and products using words, numbers, symbols, images, audio, and video" (NCTM, p. 20). Our literature review focuses on the most important factors relevant to designing such type of learning environment for the teaching of mathematics to intermediate level students.

The extant literature abounds with example of how mathematical concepts should be taught and learned. Some believe that mathematics is too technical to be learned at a distance or online, whereas others claim that online environments are more effective than in-class instructions for the teaching of some mathematical concepts. An illustration is found in the article of Schacter (1999). He describes how students who received computerized instruction in mathematics exhibit improved higher order thinking and problem-solving skills than those having regular in-class instructions. Similarly, a study by Mann *et al*. (1999) indicates that an increase in the amount of time students spent engaging in computerized instructional activities related to mathematics was associated with an increase in test scores. Meyer (2002)

points out that students learn as much [mathematics] in face-to-face courses as in high quality online courses, and that high-quality online courses are composed of high-quality educators (Russell *et al*., 2009) and design factors (Nuangchalerm, Prachagool, & Sriputta, 2011). Thus, online learning environments look promising for mathematics education.

What follows are the common literature relevant to the design and planning of technology-enabled learning environments. The literature has been conveniently grouped into different sections (and sub-sections) and appears in the following order: 'Interactions and Mathematics', 'Situated learning', 'Web-based learning environment', and finally we discuss some notable achievements of technology integration in mathematics education. Throughout the discussion, we compare and contrast the findings of our own study with those of other researchers within the review of the literature.

Factors Influencing the Teaching of Mathematics Using Technological Communication Tools

Interaction and Mathematics

Importance of Interaction

Interaction is an essential component in learning. Researchers have written extensively about the importance of online interactions in mathematics especially for students in K-12 years (DiPietro *et al*., 2008; Murphy & Rodríguez-Manzanares, 2008; Roblyer *et al*., 2007). Students are encouraged to interact with their peers since this has a positive impact on learner's achievement (Lulee, 2011). Our study reveals that students like to interact with each other. They just like to be online and to read the various posts of their peers. Some of them enjoy posting a simple note of appreciation to their peers and to show their online presence. They communicate with each other through interactions mostly in the form of written posts. Interactions happen in a constructivist classroom and students are encouraged to share their opinions and interact with each other to construct their own understanding. For example, the think-pair-share strategy is a simple known teaching strategy that teachers frequently use to get students to share their thoughts and to encourage classroom discussions. Some students learn very well through this strategy and some not so well. This whole or small group open classroom sharing may not fit everyone's learning style because not all learners are used to share their ideas on the spot with limited amount of think time. Thus, learners who do not like to share publicly or those who "prefer independent learning will find themselves more comfortable in an online learning environment" (Gagné *et al*., 2005, p. 329) as most, if not all, interactions are done in writing and this enables them to have more time to think and share what they want to share the way they want to share. Our study confirms that students who are not used to participate during face-to-face interactions have a higher level of participation in our online environment. Besides, they made thought-provoking contributions that were inexistent in the physical classroom, and this encouraged many responses from their peers. The next section highlights the importance of interaction and its influences on student's learning.

Three types of interaction

In an online math learning environment, interactions are mainly of three types, namely: learner-learner, learner-content, and learner-teacher (Anderson, 2003). In such an environment, all interactions happen via a platform which is also called 'a virtual classroom'. Our study made use of a fully online environment (see http://www.grade7trip.site90.net/) that encourages the three types of interactions. There are many factors that influence how interactions develop and evolve in an online learning setting. Some common factors are gender, learning styles, technologies, and group size (Lulee, 2011). The quality of the interactions is also important for students' learning. Deep and thoughtful contributions to discussions are much more desirable and productive than the amount of interactions (Anderson, 2003). This is precisely what we found when analyzing interactions of our participants. We noticed that interactions that were

deep and thoughtful attracted more meaningful contributions than shallow and brief interactions. In our study, the most common form of interaction that was of a high level was learner-learner, which confirms Anderson's (2003, p. 3) claim that "deep and meaningful formal learning is supported as long as one of the three forms of interaction is at a high level. The other two may be offered at minimal levels, or even eliminated, without degrading the educational experience."

Another factor of importance when evaluating interactions is the level of engagement as the amount and level of interactions happening in a learning environment do not guarantee engagement (Garrison & Cleveland-Innes, 2005; Wagner, 1994). In our study, engagement was sustained through the subject of the project as our participants had a shared interest in their project. The project was used for learning and also for brainstorming ideas. We also specified the minimum amount of interactions and adjusted the minimum amount when we realized that the interactions were not being very productive and depending on the individual needs and preferences of each group of students (see also Chen & Willits, 1998; Fahy, 2001). In teaching this course, our goal was to find the right mix between the three types of interactions (Anderson, 2003) while adjusting the amount of interactions to maximize engagement. Hence, it is important for teachers/designers to create the space to allow interactions to happen during a math class in any kind of learning environment. In addition to the learning environment, a high quality teacher with the relevant technological, pedagogical content knowledge would enhance the students' learning experience as he/she will have the knowledge and skills to adjust the course as it unfolds.

Interaction, online learning, and social presence

Students have different learning styles, types of intelligence, interests, levels of motivation, and readiness. A question that arises is whether online math would be a good choice for students who struggle in math. This category of students usually requires more frequent small group or one-on-one support from the teacher. Moreover, several authors have indicated that "learning mathematics online can be more challenging for [struggling] students due to a sense of isolation and a lack of social support in online learning environments" (Kim, Park & Cozart, 2014, p. 171). In our study, we found that to be true for students who struggle in math. They had much difficulty to interact with their peers, had little amount of posts or late posts, and some did not even complete or were late with their project. They were usually offline and the contributions they made did not attract many responses. Why such findings? Are they due to their low self-esteem in math or do they arise because of their lack of understanding and unwillingness to ask questions? Answers to these questions require further investigation, and will very likely appear in our future research agenda. Thus, it is important for designers to be aware of challenges that struggling math students will face with online learning and to provide the necessary support so that they do not feel lost and disengaged.

Another challenge for online learners is a lack of social presence that exists in online environments. Online learners learn in front of their computer in an online classroom. There is no face-to-face meeting where learners will be able to socialize and develop their friendship and know each other. There is no way to give a little pat on the shoulder as a means of encouragement. This lack of social presence that online learners face may hinder students' learning as there is less opportunity for the teacher to decipher the students' body language to know whether they are making meaning to what is happening, or whether they need help. This may cause the student to decrease their level of interactions and likely to result in the student having the tendency to sense disconnectedness (Hawkins, Barbour & Graham, 2012; Song *et al*., 2004) and subsequently, impacting students' motivation, emotions, and cognitive processes (Kozma *et al*., 2000; Schunk, Pintrich & Meece, 2008). Thus, some students will benefit more when they are in a physical classroom with face-to-face interactions whereas other students may learn very well online.

Situated Learning

Our study reveals that activities that are part of a web-based learning environment have to be situated, that is, in context and relevant to the students' prior knowledge. This is confirmed by Brown and Duguid

(2000) who claim that learning is demand driven. Additionally, Lave (1996) and Brown and Duguid (1996) suggest that learning occurs in a social setting where learners 'steal' implicit and explicit knowledge through participating and observing practitioners at the periphery or side. It was apparent in our study that the knowledgeable ones are always at the centre and learners, by being on the periphery will connect their prior knowledge to what they are experiencing, seeing, hearing to make meaning. During this process and through interactions, the learner will gradually move from the periphery to the centre to become, in Vygotsky's (1978) terms, a 'More Knowledgeable Other'. Can a learner learn mathematical concepts without understanding its applications in real life? In other words, can we teach addition, for instance, without showing to the learner its purpose in real life? If so, then it would be analogous to teaching the different manoeuvres of operating a passenger vehicle but not combining them for the learner to understand how all the parts fit in together to drive a vehicle. Learning mathematics has some similarities to driving a car. All concepts that are learned by the learner fit together to make meaning to what are around them and enable them to solve everyday problems. Even though an individual may be working on a task individually or in isolation, learning is inherently social, or sociocultural, in that it incorporates socially evolved and socially organized tools (Cole and Wertsch, 1996). Thus, all activities that are part of a learning environment have to be in a context and situated for the learners to make meaning to what they are learning and how all the concepts fit in together in real life.

Web Based Learning Environment

Three types of web-based learning environment

There are three types of web-based learning environment commonly used in education. They serve different purposes and are set up with different functionalities. The choice of the type of environment will depend on the needs of the instructor/teacher and to the extent that it will serve. The first type of environment is an environment where the user is a passive recipient of information. This learning environment may be in the form of a website and does not allow any input from the user. The second type allows some upload of information from the user. It is usually partly interactive and can be asynchronous or synchronous. An example is an e-commerce website or a search engine where there may be some restrictions on what can be inputted and shared. The third type of environment is one that is truly interactive and that allows diverse uses. Under the latter category, an online platform that allows users to interact with other users synchronously or asynchronously is common. The platform may include the use of audios, videos, chat rooms, forums, emails, and instant communications, and the main goal is to provide users with a complete and true learning experience that can mimic or surpass in-class instruction. This is the type of environment that is more commonly used by school boards, colleges and universities for educational purposes. Our study made use of this type of environment and incorporated hypermedia, which is mentioned later. However, one caveat to online learning is that the teacher/designer has to be quite knowledgeable and skilled when planning and designing such type of environment. The environment and its components will be somewhat different if used for intermediate math students compared to graduate students. So, having the technological, pedagogical, and content knowledge is important to designing such environment. The next section describes some of the major considerations when designing such learning environment.

Design considerations for an online learning environment

There are important factors to consider when thinking of a vibrant online learning environment. Whether the learning environment is of the first, second or third type, the factors to consider during the planning and designing stage will vary depending on the content, students, teacher, and available infrastructure. We referred to three frameworks when planning and designing the online learning environment used in this study.

The first one we used is from Hung and Chen (2001) who propose a comprehensive framework with four components, namely, 'situatedness', 'commonality', 'interdependency', and 'infrastructure'. They recommend that the activities that are part of the learning environment be contextualised and situated; that tasks and projects to be relevant to the demands, interests and needs of the students; that the environment is conducive to collaboration where students feel dependent on each other's strengths; and that there is an appropriate infrastructure that promotes accountability, and sustains motivation. In our study, we planned all the activities around raising funds for the participants' upcoming grade 8 graduation trip. We have set up an interactive website that includes hypermedia and the participants were guided to complete all activities at home by interacting with other learners and their teacher via the online environment. They were very engaged and focused on what they were learning, as the online course/task was properly planned with clear expectations and guidelines (see also Dykman & Davis, 2008; Ku *et al*., 2011), and clear assignment rubrics given (see also Lee, 2014). Accordingly, with an environment that is well designed, it is likely that this has resulted into an increase in student's level of satisfaction with their online learning experiences (Lee, 2014), an aspect that should be taken into account when designing courses and building online environments (Zhu, 2012).

The second framework we used is from Sawyer (2008), from the Learning Sciences Group, who posits that the most effective learning environments have to provide (a) customised learning for every child so that learners with all types of intelligence and learning style are served; (b) the possibility for learners to access knowledge from diverse sources; (c) activities that require learners to collaborate and construct their own knowledge and understanding, and (d) opportunities for teachers to give feedback to students at different intervals. Our learning environment encourages learners to access the internet for additional information and one of the activities requires them to sign up for a free Skype account. They also had to collaborate and to interact with each other at different intervals.

The third framework we made reference to is from Boettcher's (2007) ten principles, derived from brain research. The main points of this framework are listed below:

1. Every structured learning experience has four elements with the learner at the center
2. Every learning experience includes the environment in which the learner interacts
3. We shape our tools and our tools shape us
4. Faculty are the directors of the learning experience
5. Learners bring their own personalized knowledge, skills, and attitudes to the learning experience
6. Every learner has a Zone of Proximal Development that defines the space that a learner is ready to develop into useful knowledge
7. Concepts are not words; concepts are organized and intricate knowledge clusters
8. All learners do not need to learn all course content; all learners do need to learn the core concepts
9. Different instruction is required for different learning outcomes
10. Everything else being equal, more time-on-task equals more

We used the three frameworks during the design phase of our learning environment. We attempted to find the right mix of components and activities to include so as to encourage deep interactions and to sustain a high level of student engagement. We amalgamated the different recommendations from the three framework mentioned, together with our understanding of teaching and learning of intermediate students and math. One of the authors facilitated the teaching and online learning of math with the participants. It seems reasonable to say that a well-designed online math learning environment helps learners to experience great learning. Moreover, a knowledgeable and insightful facilitator/teacher equally plays a significant role in an online environment.

Hypermedia in an online learning environment

The use of hypermedia greatly enhances a learner's online experience. Hypermedia consists of sound, graphics, text and videos. A learner's experience is greatly improved by the presence of hypermedia as more senses are triggered when more than texts are used to present a concept. The effectiveness of the use of hypermedia is documented by Renninger *et al*. (2011, p. 235) who found that, following a study

conducted with teachers in an online Teacher Professional Development course, "the potential of using hypermedia for online learning … is significant." However, this success is dependent "to the extent that it addresses learners' different levels of interest, self-efficacy, and disciplinary knowledge, in this case, teachers learning how to teach mathematics" (Renninger *et al*., 2011, p. 246) through an online learning environment. The math online learning environment used in our study includes videos, images, internet links to additional resources, texts, and graphics.

Some Successes with Online Mathematics

Learning math online has proven to be successful with elementary students in various contexts. In some cases, it resulted in deeper understanding and higher academic achievement than in-class instructions. Some of the most known successes with teaching math through an online learning environment are summarized below:

- Hughes *et al.* (2007) explore Algebra achievement of secondary school students in both online and traditional mathematics classrooms and the perception of students towards both classroom ambiances. Their findings show that students' scores on an Algebra understanding achievement test were higher in online courses than the scores in traditional courses.
- Maloy, Edwards & Anderson (2010) report that the web-based mathematics tutoring system 4MALITY cause a mean gain of around 25% in test scores from pre-test to post-test in the Massachusetts Comprehensive Assessment System (MCAS) math grade 4 exams when administered to 125 Massachusetts fourth graders on 3 rural school districts in 2007-2008.
- Several authors conclude that some mathematical processes such as learning multiplication facts and fractions have been found to be easier for students when using computer-based instruction (Irish, 2002; Schacter, 1999; Tienken & Wilson, 2007).
- Clark and Whetstone (2014, p. 465) report that the online tutoring software Math Whizz administered to 2,542 students from 15 elementary schools in the state of Kentucky showed that "50 minutes of Math Whizz per week could result in three quarters of a year's worth of improvement in mathematics ability." In their study, the average weekly usage of Math Whizz is around 23 minutes, and this results in an average improvement of 0.4 years' worth of improvement in mathematics ability.

Implications for Online Mathematics Education

Mathematics is a subject that has been traditionally taught in a face-to-face classroom, that is, through a medium of instruction that has had great success and that is widely accepted and approved. This has been like that since centuries ago. However, with the rapid proliferation of technology among teenagers, many schools are offering online courses to meet the needs of their students. Transitioning from in-class math to online math may prove challenging for teachers and students because it is widely accepted that math is a subject that requires a lot of teacher support and face-to-face communication. However, the successes of teaching math through online learning have been documented by several researchers, and in some cases, have proven to be more successful than in-class instructions for some specific math concepts. In our study, we have found that the inclusion of short videos on our website enables students to review while they are completing their tasks. Some of them reviewed the videos several times for more understanding. In a physical classroom scenario, it would have been challenging or almost impossible for the teacher to explain the same process several times to some students due to time constraint.

The availability of web links to other websites really helped our participants to get a better and wider understanding of the different concepts under study as they were able to read, understand and complete their project at their own pace. All what they require for their project is available on the website and online and they can refer to anytime they want to. This is very useful for students who struggle with

memorizing and understanding concepts. Furthermore, all the learners do not fear of being judged because they asked a question that was already answered by the teacher. In our website, we have planned to have students interact at some specific point in the project, especially where there are complex concepts to be understood.

Students have different learning styles, types of intelligence and require different amount of time to learn and understand. We found that some students learn better online than in class. In our study, some students reported that they learn better than in class because they had time to review the online posts and learned a lot through the posts. They also mentioned that, unlike in a physical classroom, every interaction is posted and accessible days or weeks after the discussions are done. They mentioned that they went back to review if they forgot what was discussed. They were also able to choose what discussion thread they wanted to participate and had time to think and decide on what and how to share, which is contrary to in-class interaction which is often spontaneous.

In the light of our findings pertaining to teaching mathematics through a fully interactive online learning environment that makes use of common technological tools, we have come up with a set of considerations for designing an online math learning environment. This is supported and informed by our extensive review of the literature in the field. We have also reported elsewhere some of the findings of the present study (see Hung Cheong Lan and Ah-Teck, 2014).

The online learning environment for teaching intermediate mathematics education should:

- Contain some videos (hypermedia) that explain difficult concepts and how-tos. There have to be some links to external resources for those who need additional support.
- Provide a space for learners to communicate their ideas and engage in discussions in writing in small and large group. This space may be through the use of a platform such as Moodle, Pepper or Blackboard.
- Enable the three types of interaction
    - Learner to Learner(s) (sharing and working collaboratively on a product and sharing to large group discussion)
    - Learner to Teacher (support from the teacher)
    - Learner to Content (through web pages, short videos, external links, etc.)
- Include the use of authentic context-based math activities that emanate from real life situations so as to sustain the learner's engagement.
- Include at least an activity that requires every learner to collaborate with at least another learner. The pair or small group is to be formed by the teacher or by the students.
- Provide a space where the teacher can interact with the learners in a synchronous way and provides support individually or to a group of learners. Communication software like Skype, Go-To-Meeting, Adobe Connect, etc. enables users to see each other via a webcam and to share their screen for deeper discussions. This increases the social presence aspect as users will be able to use their body language while communicating.
- Enable the teacher to intervene to clarify any misunderstandings and redirect discussions if need be.
- Specify the minimum number of interactions that are expected so that learners know how much is enough.
- Specify the time-lines by which learners have to post their work and post comment for others to respond, and by when they are expected to respond. This enables more timely and focused interactions.
- Provide learners the possibility to choose to which discussion thread they want to participate.

Closing Comments

To conclude, it is reasonable to assume that teaching and learning math virtually is no longer a myth. It is also realistic to say that online math learning may be very effective with elementary students, given that

several studies conducted in the K-12 school system in the United States have shown a positive impact on students' achievement in math when technology is integrated and supplemented in the daily classroom. A learning environment that is well designed and facilitated by a teacher with a high level of technological, pedagogical, and content knowledge will likely produce deep learning and high student achievement.

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